Quantum Force in Superconductor

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Transitions between states with continuous (called as classical state) and discrete (called as quantum state) spectrum of permitted momentum values is considered. The persistent current can exist along the ring circumference in the quantum state in contrast to the classical state. Therefore the average momentum can changes at the considered transitions. In order to describe the reiterated switching into and out the quantum state an additional term is introduced in the classical Boltzmann transport equation. The force inducing the momentum change at the appearance of the persistent current is called as quantum force. It is shown that dc potential difference is induced on ring segments by the reiterated switching if the dissipation force is not homogeneous along the ring circumference. The closing of the superconducting state in the ring is considered as real example of the transition from classical to quantum state.

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Superconductivity is a macroscopic quantum phenomenon. This means that some macroscopic effects observed in superconductors can not be described by classical mechanics. First of such effects was discovered in 1933 year by W.Meissner and R.Ochsenfeld [1]. The Meissner effect is caused by the quantization of the generalized momentum $p = mv + (q/c)A$ of a quantum particles along a closed path l [2]. Because $\int_l dlp = n2\pi\hbar$ the value $\int_l dlA + (mc/q)\int_l dlv = \Phi + (mc/q)\int_l dlv_s$ called by F.London [2] as fluxoid is quantized: $\Phi + (mc/q)\int_l dlv = \Phi_0 n$ [3]. A is the vector potential; $\Phi = \int_l dlA$ is the magnetic flux contained within the closed path l; $\Phi_0 = 2\pi\hbar c/q$ is the flux quantum. The charge of the superconducting pair $q = 2e$. Therefore $\Phi_0 = \pi\hbar c/e$ for superconductors [4].

The magnetic flux is expelled from the interior of a superconductor because the superconducting pairs are condensed bosons with the same n value. When the wave function of the superconducting pairs does not have any singularity inside l, it can be tightened to point without crossing of a singularity. Therefore the n value and consequently the fluxoid value should be equal zero $\Phi + (mc/e)\int_l dlv_s = \Phi_0 n = 0$, i.e. in the interior of the superconductor, where $v_s = 0$, the magnetic flux Φ should be equal zero.

When the l is a closed path along the ring circumference the n can be any integer number. When the ring wall is wide, $w \gg \lambda$, the flux quantization $\Phi = \Phi_0 n$ [5] takes place because $v_s = 0$ in the interior of the superconductor. Here λ is the penetration depth of magnetic field. The quantization of the magnetic flux was observed first in [6]. In a superconducting ring (or tube) with narrow wall the velocity of superconducting pairs is quantized

$$\int_l dlv_s = \frac{\hbar}{Rm}(n - \frac{\Phi}{\Phi_0}) \quad (1)$$

This quantization was manifested first by the Little-Parks experiment [7]. The limit case of the ring with narrow wall ($w \ll R, \lambda$) is considered in the present paper.

The nonzero average velocity in the thermodynamic equilibrium state means the existence of the persistent current $j_{p.c.}$ because $j = qn_q < v >$ [8]. Here n_q is the density of particles with charge q. There is a principle difference between classical and quantum mechanics. The persistent current is equal

$$j_{p.c.} = q\sum_p vf_0 = \frac{q}{m}\sum_p (p - \frac{q}{c}A)f_0 \quad (2)$$

The distribution function of the equilibrium state, f_0, is even function of the velocity because it depends only on the relation E_p/k_BT and the kinetic energy E_p is proportional to v^2 in a consequence of the space symmetry: $E_p = mv^2/2 = (p - (q/c)A)^2/2m$.

When the spectrum of permitted p values is continuous the summation in (2) can be replaced by the integration and $p = mv + (q/c)A$ can be replaced by $p = mv$. $\int_{-\infty}^{\infty} dp\ pf_0 = 0$ because f_0 is even function of $p = mv$. Consequently, the persistent current can exist only in states with discrete p spectrum. According to the classical mechanics all p values are permitted. The discrete spectrum is characteristic feature of the quantum mechanics. Therefore the state with continuous p spectrum will be called in this paper as classical state and the one with discrete spectrum as quantum state. The summation in (2) can be replaced by the integration when the energy difference between adjacent permitted states $E_{n+1} - E_n$ is small in comparison with k_BT therefore the p spectrum may be considered as continuous at $(E_{n+1} - E_n)/k_BT \ll 1$.

The persistent current $j_{p.c.} = (q/m)\sum_p (p - q\Phi/cl)f_0 = (c\Phi_0/\lambda^2 l)\chi$ can exist in a ring when the p spectrum along the ring circumference $l = 2\pi R$ is discrete. Here $\lambda = (mc^2/n_{ef}q^2)^{1/2}$ is an analog of the London penetration depth; n_{ef} is an effective density; $\chi = \chi(\Phi/\Phi_0)$ is a periodic function of the magnetic flux Φ contained within the ring. $-05 < \chi < 0.5$, $\chi = 0$ at $\Phi/\Phi_0 = n$ and $\Phi/\Phi_0 = n + 0.5$. In the case of a homogeneous superconducting ring $n_{ef} = n_s$ is the density of

supercondicting pairs and λ is the London penetration depth.

The persistent current can be not equal zero for all statistics: Bose-Einstein, Fermi-Dirac and classical. But in normal metal it can observed only in a perfect mesoscopic ring with small radius at very low temperature. Electrons have discrete spectrum of momentum along the ring circumference $p_n = (\hbar/R)n$ when it's mean free path $l_{f.p.} \gg l$. In the opposite limit, $l_{f.p.} \ll l$, electron may be considered as classical particles having at the same time a coordinate and a momentum [9]. The momentum uncertainty in this case $\Delta p > \hbar/\Delta x > \hbar/l_{f.p.}$ exceeds the momentum difference between adjacent permitted states $p_{n+1} - p_n = (\hbar/R)$, i.e. the spectrum is continuous. Therefore the persistent current can exist only at $l_{f.p.} \gg l$ and at very low temperature because for one electron $E_{n+1} - E_n \simeq \hbar^2/R^2 2m \simeq k_B T$ at $T = 1\ K$ and $R = 0.6\ \mu m$.

In a superconducting ring $l_{f.p.} = \infty$ and $E_{n+1} - E_n \simeq sln_s\hbar^2/R^2 2m \gg \hbar^2/R^2 2m$ because superconducting pairs are condensed bosons. sln_s is the number of superconducting pairs in the ring; s is the area of ring wall section. Therefore superconductivity is a macroscopic quantum phenomenon. The persistent current can exist in superconductor with big radius. In a homogeneous superconducting ring $j_{p.c.} = 2e(\hbar/Rm)n_s\chi$, where $\chi \simeq n - \Phi/\Phi_0$ at $sln_s\hbar^2/R^2 2m \gg k_B T$. Here the n is conformed with minimum permitted velocity, i.e. $-0.5 < n - \Phi/\Phi_0 < 0.5$ [3].

The possibility of the persistent current was pointed out first for the Bose-Einstein condensation [10]. Such current is observed both in superconductors and superfluids (see [11]). The rotation of the reservoir with superfluid takes the place of the magnetic flux Φ at $q = 0$ [11]. The persistent current of electrons in normal metal mesoscopic systems was predicted first by Kulik [12] and later rediscovered by Buttiker et al. [13]. This predictions was confirmed by experimental result [14]. Now this phenomenon is in progress to study in different aspects [15].

The division of classical and quantum states used in this paper is relative. For example, quantum effects, such as the interference effect [16], are observed at short elastic mean free path and long inelastic mean free path. Although the p spectrum is continuous at short elastic mean free path. The persistent current is reduced with the increase of both inelastic and elastic scattering [17].

Thus, according to our modern knowledge the equilibrium states of a mesoscopic ring may be distinguished by such macroscopic parameter as the persistent current: $\sum_p qvf_{cl} = 0$ in the classical state with continuous p spectrum and $\sum_p qvf_{qu} = j_{p.c.}$ in the quantum state with discrete p spectrum. Here f_{cl} is the distribution function of the equilibrium classical state and f_{qu} is the distribution function of the equilibrium quantum state. The object of the present paper is the consideration of reiterated transitions between f_{cl} and f_{qu}.

The relaxation of the persistent current after the transition to the classical state (from f_{qu} to f_{cl}) can be described by the Boltzmann transport equation [9]

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial l} + qE\frac{\partial f}{\partial p} = -\frac{f_1}{\tau} \qquad (3)$$

τ is the mean time between collisions; $f_1 = f - f_0$ is the deviation of the distribution function from the equilibrium state ($f_0 = f_{cl}$ in this case). One-dimensional case is considered: f changes only along the ring circumference.

In order to describe this process we do not must exceed the limits of the classical mechanics. The disappearance of a current in a consequence of the dissipation is usual process of the classical mechanics. Some assumptions must be made in the derivation of the transport equation (3) [18]. First of all it is the "random phase" assumption. It is assumed these assumptions are valid in the classical state.

The appearance of the persistent current contradicts to the classical mechanics. But the transport equation can be used for the phenomenological description of the transition from f_{cl} to f_{qu} if a new term $\aleph$ is added to it:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial l} + qE\frac{\partial f}{\partial p} = \sum_{t_q} \aleph(t - t_q) - \frac{f_1}{\tau} \qquad (4)$$

$\aleph(t - t_q)$ is not equal zero only during a time interval from $t = t_q$ to $t = t_q + \Delta t_q$ when the transition from the classical to quantum equilibrium state takes place. According to (4) $\int_{\Delta t_q} dt\aleph = \int_{\Delta t_q} dt df/dt + \int_{\Delta t_q} dt f_1/\tau = f_{qu} - f_{cl} + \int_{\Delta t_q} dt f_1/\tau$.

The generalized momentum along the ring circumference changes on $\Delta P = (m/q)j_{p.c.}[1 + (L/l)(s/\lambda^2)]$ at the transition between the classical and quantum states. According to the classical mechanics any momentum change is caused by a force. The balance on the average forces

$$\frac{\partial P}{\partial t} - F_p - F_e = \sum_{t_q} F_q(t - t_q) - F_{dis} \qquad (5)$$

is obtained by multiplication of the transport equation (4) by the momentum and summing up the p states. $P = \sum_p pf$; $F_p = -\partial(\sum_p pvf)/\partial l = -\partial(n_q < pv >)/\partial l$ is the force of the pressure; $F_e = -qE\sum_p p\partial f/\partial p = qEn_q$ is the force of the electric field; $F_{dis} = \sum_p pf_1/\tau$ is the dissipation force. $F_q = \sum_p p\aleph$. $\int_{\Delta t_q} dtF_q = \int_{\Delta t_q} dt\sum_p p\aleph = \sum_p pf_{qu} - \sum_p pf_{cl} + \int_{\Delta t_q} dt\sum_p pf_1/\tau = (m/q)j_{p.c.}[1+(L/l)(s/\lambda^2)] + \int_{\Delta t_q} dtF_{dis}$. The F_q describes the momentum change at the transition to the quantum state. Therefore it is natural to call it as quantum force.

After the full cycle: $f_{cl} \rightarrow f_{qu} \rightarrow f_{cl}$, the momentum P remains invariable, $\int_{t_{cyc}} dt\partial P/\partial t = 0$. Consequently, according to (5) $\int_{t_{cyc}} dt(F_p + F_e + F_q -$

$F_{dis}) = 0$. $\int_l dlF_p = -\int_l dl\partial(n_q < pv >)/\partial l = 0$; $\int_{t_{cyc}} dt \int_l dlF_e = qn_q \int_{t_{cyc}} dt \int_l dlE = -(qn_q/c)\int_{t_{cyc}} dtd\Phi/dt = (qn_q/c)(\Phi - \Phi) = 0$. Consequently $\int_{t_{cyc}} dt \int_l dlF_q = \int_{t_{cyc}} dt \int_l dlF_{dis}$.

Thus, the quantum force is opposed to the dissipation force. There is important difference between the quantum and dissipation forces in a consequence of the uncertainty in space. The quantum force can not be localized in time and in space in a consequence of the uncertainty relations: $\Delta E \Delta t > \hbar$ and $\Delta p \Delta l > \hbar$. The uncertainty in time is not important here because the dissipation force is the average value of the forces acting at chaotic collisions. But the dissipation force can be localized in space. Whereas the quantum force can not be localized along the ring circumference in principle because a particle should be spread all over ring in the quantum state: $\Delta l > \hbar/\Delta p \gg \hbar/(p_{n+1} - p_n) = R$.

The dissipation $\int_{\Delta t_q} dtF_{dis}$ during the transition from f_{cl} to f_{qu} is weak when the time Δt_q is long. In this case (which is considered in the present paper) the quantum force $\int_{\Delta t_q} dtF_q \simeq (m/q)j_{p.c.}[1 + (L/l)(s/\lambda^2)]$ is constant along the ring circumference because $I_{p.c.} = sj_{p.c.}$ should be constant in the equilibrium state (it is proposed s is constant). Therefore, $\int_{t_{cyc}} dtF_q - \int_{t_{cyc}} dtF_{dis} \neq 0$ and consequently $\int_{t_{cyc}} dtF_p + \int_{t_{cyc}} dtF_e \neq 0$ in an inhomogeneous ring, in which F_{dis} is not constant along the ring circumference.

Both F_p and F_e are induced by deviation of the electron density Δn_q from the equilibrium value ($\Delta n_q \ll n_q$). In the order of value $F_p \approx - < pv > \Delta n_q/\Delta l$ and $F_e \approx q^2 n_q \Delta n_q \Delta l = q^2/n_q^{-1/3}(\Delta l/n_q^{-1/3})^2 \Delta n_q/\Delta l$. The characteristic length Δl of Δn_q change is much longer than the distance between electrons: $\Delta l \gg n_q^{-1/3}$. In any metal $< pv >\approx q^2/n_q^{-1/3}$ [9]. Consequently, $F_p \ll F_e$ and $\int_{t_{cyc}} dtF_e \simeq \int_{t_{cyc}} dtF_{dis} - \int_{t_{cyc}} dtF_q$

This means that the reiterated switching into and out the quantum state can induced the voltage in segments of inhomogeneous ring the average value of which by a long time T_{long}, $< E >\simeq (< F_{dis} > - < F_q >)/qn_q$, is not equal zero. The dc potential difference on a segment l_a is equal $V_a = l_a < E >\simeq -l_a < F_q > /qn_q$ when $< F_{dis} >= 0$ in this segment. $< F_q >= (\int_{T_{long}} dtF_q)/T_{long} = (\sum_{t_q} (m/q)j_{p.c.}(q)[1 + (L/l)(s/\lambda^2)])/T_{long} = (m/q) < j_{p.c.} > [1+(L/l)(s/\lambda^2)]f$. Consequently,

$$V_a = -\frac{l_a}{l} \frac{< n_{ef} >}{n_q} \frac{\Phi_0 f}{c} (1 + \frac{L}{l}\frac{s}{\lambda^2})\chi \qquad (6)$$

$< j_{p.c.} >= \sum_{t_q} j_{p.c.}/N_l$ is the average persistent current in the quantum states; $f = N_l/T_{long}$ is the average frequency of the switching; N_l is the number of the switching during the time T_{long}. The potential difference on other segment l_b, $V_b = -V_a$ because $\int_l dl < E >= 0$, $(l_a + l_b = l)$.

The closing of the superconducting state in the ring is the real example of the transition from classical to quantum state. The superconducting current $j_s = 2en_s v_s$ should be constant along the ring circumference in the equilibrium state. Therefore the kinetic energy of superconducting pair in the ring is equal $E_p = s\int_l dln_s 2mv_s^2/2 = (sj_s/4e)\int_l dl2mv_s = (sj_s 2\pi\hbar/4e)(n - \Phi/\Phi_0)$ (here the velocity quantization (1) is taken into account). According to (1) $j_s = (e\hbar/Rm < n_s^{-1} >)(n - \Phi/\Phi_0)$, where $< n_s^{-1} >= l^{-1}\int_l dln_s^{-1}$. Consequently, $E_p = (s2\pi\hbar^2/4Rm < n_s^{-1} >)(n - \Phi/\Phi_0)^2$ and the energy of the magnetic flux induced by the superconducting current $E_L = LI_s^2/2c^2 = (Ls^2e^2\hbar^2/2c^2R^2m^2 < n_s^{-1} >^2)(n - \Phi/\Phi_0)^2 = (Ls/l\lambda_0^2)n'_s E_p$. Here $\lambda_0 = (c^2 2m/4e^2 n_s(0))^{1/2}$ is the London penetration depth at $T = 0$; $n'_s = (n_s(0) < n_s^{-1} >)^{-1}$; $n_s(0)$ is the density of superconducting pairs at $T = 0$.

In a strongly inhomogeneous ring in which the density n_s in a segment l_b is much lower that in other segment l_a, $< n_s^{-1} >\simeq l^{-1}\int_{l_b} dln_s^{-1} \approx l_b/ln_{sb}$. There can be taken into account the Josephson current if n_{sb} is considered as an effective density. The energy difference between adjacent permitted states $E_{n+1} - E_n \simeq s2\pi\hbar^2/4Rm < n_s^{-1} >\approx s\pi^2\hbar^2 n_{sb}/l_b m$ is determined by lowest density n_{sb}. Therefore the p spectrum may be considered as continuous when the superconducting state in the ring is not closed, i.e. when $n_{sb} \approx 0$ and consequently $(E_{n+1} - E_n)/k_B T \ll 1$. The "random phase" assumption is valid in this case. At large n_{sb} value $(E_{n+1} - E_n)/k_B T \gg 1$. Thus, the persistent current appears at the closing of the superconducting state and disappears at the break of phase coherence along the ring circumference.

The quantum force accelerates the superconducting pair in the l_a segment from zero to $v_s = (2\pi\hbar/m)(n_{sb}/(l_a n_{sb} + l_b n_{sa}))(n - \Phi/\Phi_0)$ against the force of the electric field $F_e = -n_{sa}(2e/c)LdI/dt$. Here n_{sa} and n_{sb} are average density of superconducting pairs in the segment l_a and l_b. The electric force acts both on superconducting pairs and normal electrons. Whereas the quantum force acts only on superconducting pairs and the dissipation force acts only on normal electrons. Therefore the balance on the average forces $< F_e >= 2en_{sa} < E > +en_e < E >\simeq < F_{dis} > - < F_q >$ falls apart into the balance of forces acting on superconducting pairs $2en_{sa} < E >\simeq - < F_q >$ and on normal electrons $en_e < E >\simeq < F_{dis} >$. It is assumed that the densities of superconducting pairs n_{sa} and normal electrons n_e in the l_a segment are not change.

At $(E_{n+1} - E_n)/k_B T \gg 1$, $j_{p.c.} = j_s = (4e\pi\hbar/m)(n_{sa}n_{sb}/(l_a n_{sb} + l_b n_{sa}))(n - \Phi/\Phi_0)$ with the n value conformed with minimum permitted velocity, i.e. $-0.5 < n - \Phi/\Phi_0 < 0.5$. At $(E_{n+1} - E_n)/k_B T \ll 1$, $j_{p.c.} = 0$. Consequently, the dc potential difference $V_a = (l_a n_{sbm}/(l_a n_{sbm} + l_b n_{sa}))[1 + (L/l)(s/\lambda^2)]\chi\Phi_0 f/c$ is induced on the superconducting segment l_a by the reiterated switching of the n_{sb} from a large value n_{sbm} to

$n_{sb} \approx 0$ with the average frequency f.

Thus, the inhomogeneous superconducting ring can be considered as dc generator. When the n_{sb} change is induced by temperature change then the heat energy is transformed in electric energy. The superconducting state can be also interrupted and closed by mechanical interrupting and closing of the ring. In this case the mechanical energy can be transformed to the electric energy. The superconducting ring can not substitute traditional generators because the voltage induced in it is very weak. Because $l_a n_{sbm}/(l_a n_{sbm} + l_b n_{sa}) < 1$, $1 + (L/l)(s/\lambda^2) \approx 1$ and $|\chi| < 0.5$ the maximum voltage $V_a \simeq \Phi_0 f/2c \simeq 10^{-9} f\ \mu V$. But the consideration of this energy transformation gives important physical results.

According to the classical mechanics, in order to change energy value a work should be done. The work done by the quantum force produces the kinetic E_p and magnetic flux E_L energy:

$$E_p + E_L = s\frac{\pi\hbar^2}{2Rm}\frac{1}{< n_s^{-1} >}(1 + \frac{L}{l}\frac{s}{\lambda_0^2}n'_s)(n - \frac{\Phi}{\Phi_0})^2 \quad (7)$$

. Both these energies increase the energy of the superconducting state at $\Phi/\Phi_0 \neq n$. The Little-Parks experiment [7] is explained [19] by the periodical dependence E_p on Φ. The energy of magnetic flux does not influence on the value of the critical temperature because the E_L is proportional to n_s^2 ($< n_s^{-1} >= 1/n_s$ in a homogeneous ring). The Little-Parks effect in an inhomogeneous superconducting ring is considered in [20].

We may say that the critical temperature is reduced at $\Phi/\Phi_0 \neq n$ because the quantum force should be overcome at the transition to the superconducting state. This explanation does not differ from the Tinkham's explanation [19] and does not give any new knowledge because the critical temperature is scalar. The introducing of the quantum force is useful at the consideration of vector quantities, such as the voltage.

The work W is the product of a force F and a path dx: $W = \int dxF$. It is impossible to point out a path when the closing of the superconducting state is induced by temperature change. But the real path exists when the mechanical closing of the superconducting ring interrupted by Josephson junction takes place. The Josephson current decreases exponentially with increasing of break width b and has ignorable value when b exceeds some nanometers [21]. Therefore the energy $E_p + E_L$ increases from zero to the value determined by the relation (7), $E_p + E_L \approx E_p \approx (s/\lambda^2)(\Phi_0^2/4\pi R)(n - \Phi/\Phi_0)^2$, when the b value is changed from some nanometers to zero. This means: in order to close the ring the work $W \simeq E_p + E_L$ should be expended, i.e. the quantum force, the average value of which is equal $W/\Delta b$ should be overcome. At $s/\lambda^2 \simeq 1$, $n - \Phi/\Phi_0 = 1/2$ and $R = 1\mu m$, $E_p \approx 3\ 10^{-20}\ J$. Consequently, at $\Delta b = 10\ nm$, $W/\Delta b \approx 3\ 10^{-20}\ N$. It is very weak force. But there is important to show a connection of the quantum force with a real classical force. This consideration shows that the wave function can have an elasticity.

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